# MATH 5635: Stochastic Calculus for Finance I

#### Course Outline

#### Autumn 2025

Instructor: Dr. Kenneth Ng E-mail: ng.499@osu.edu

Office: Room 440, Mathematics Tower (MW 440)

Class Hours and Location: 4:10 - 5:05 pm, Mon, Wed, Fri; Enarson Classroom Bldg 245

Office Hours: Monday and Friday, 3:00 - 4:00 pm, or by appointment

## Course Description

This course provides a mathematical foundation to advanced financial modelling and derivative pricing. The first part of the course covers mathematical tools in abstract probability theory, Lebesgue integrals, and change of measures. The second part of the course covers martingale theories, Itô's calculus, stochastic differential equations, and Girsanov's theorem. The third part of the course covers risk neutral measures, Black-Scholes option pricing, and Greeks. This course can be followed by MATH 5636 Stochastic Calculus for Finance II.

## Course Materials

- The class notes available on Canvas would be the main source of course materials.
- Reference books:
  - 1. Stochastic Calculus for Finance II: Continuous-Time Models by Steven Shreve.
  - 2. Stochastic Differential Equations: An Introduction with Applications by Bernt Øksendal.
  - 3. Brownian Motion and Stochastic Calculus by Ioannis Karatzas and Steven Shreve.
  - 4. Probability Essentials by Jean Jacod and Philip Protter.
  - 5. Probability: Theory and Examples by Rick Durrett.

# Prerequisites

A grade of C- or above in 3589 or 3345; and a grade of C- or above in 4530, 5530H, or STAT 4201; and enrollment in Math major or Actuarial Science major; or Grad standing; or permission of department.

# Grading and Assessments

- Grade composition:
  - 1. Homework: <u>40%</u> of the final grade. About 5-8 homework assignments, one for each chapter. Each homework carries equal weight.
  - 2. One midterm exam: 30% of the final grade.
  - 3. Final exam: 30% of the final grade.
- Grades of this course will be assigned mainly based on the OSU Standard Grade Scheme. Mild adjustment could be made based on difficulties of the exam papers. For students' reference, the OSU Standard Grade Scheme is given below:

Grade	Score
A	93%
A-	90%
B+	87%
В	83%
В-	80%
$\mathrm{C}+$	77%
С	73%
C-	70%
D+	67%
D	60%
E	Below 60%

# Course Policies

#### Homework Submission Policies

Please submit your assignment as a single PDF file to Canvas. The due dates of homework will be specified on Canvas, which will usually be two weeks after the homework was posted.

Late assignments will be accepted for no penalty if a valid excuse is communicated to the instructor before the deadline. Assignments will be accepted with a 10%/30%/50% deduction within 1/2/3 days after the deadline. After this any assignments handed will be given 0.

### Academic Integrity and Honesty

Academic integrity is essential to maintaining an environment that fosters excellence in teaching, research, and other educational and scholarly activities. Thus, The Ohio State University and the Committee on Academic Misconduct (COAM) expect that all students have read and understand the University's Code of Student Conduct, and that all students will complete all academic and scholarly assignments with fairness and honesty. Students must recognize that failure to follow the rules and guidelines established in the University's Code of Student Conduct and this syllabus may constitute Academic Misconduct.

The Ohio State University's Code of Student Conduct (Section 3335-23-04) defines academic misconduct as: Any activity that tends to compromise the academic integrity of the University or subvert the educational process. Examples of academic misconduct include (but are not limited to) plagiarism, collusion (unauthorized collaboration), copying the work of another student, and possession of unauthorized materials during an examination. Ignorance of the University's Code of Student Conduct is never considered an excuse for academic misconduct, so please review the Code of Student Conduct and, specifically, the sections dealing with academic misconduct.

If an instructor suspects that a student has committed academic misconduct in this course, the instructor is obligated by University Rules to report those suspicions to the Committee on Academic Misconduct. If COAM determines that a student violated the University's Code of Student Conduct (i.e., committed academic misconduct), the sanctions for the misconduct could include a failing grade in the course and suspension or dismissal from the University.

If students have questions about the above policy or what constitutes academic misconduct in this course, they should contact the instructor.

## Artificial Intelligence and Academic Integrity

There has been a significant increase in the popularity and availability of a variety of generative artificial intelligence (AI) tools, including ChatGPT, Sudowrite, and others. These tools will help shape the future of work, research and technology, but when used in the wrong way, they can stand in conflict with academic integrity at Ohio State.

All students have important obligations under the Code of Student Conduct to complete all academic and scholarly activities with fairness and honesty. Our professional students also have the responsibility to uphold the professional and ethical standards found in their respective academic honor codes. Specifically, students are not to use unauthorized assistance in the laboratory, on field work, in scholarship, or on a course assignment unless such assistance has been authorized specifically by the course instructor. In addition, students are not to submit their work without acknowledging any word-for-word use and/or paraphrasing of writing, ideas or other work that is not your own. These requirements apply to all students undergraduate, graduate, and professional.

To maintain a culture of integrity and respect, these generative AI tools should not be used in the completion of course assignments unless an instructor for a given course specifically authorizes their use. Some instructors may approve of using generative AI tools in the academic setting for specific goals. However, these tools should be used only with the explicit and clear permission of each individual instructor, and then only in the ways allowed by the instructor.

#### Accommodations for Disabilities

The university strives to maintain a healthy and accessible environment to support student learning in and out of the classroom. If you anticipate or experience academic barriers based on your disability (including mental health, chronic, or temporary medical conditions), please let me know immediately so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services (SLDS). After registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion.

If you are ill and need to miss class, including if you are staying home and away from others while experiencing symptoms of a viral infection or fever, please let me know immediately. In cases where illness interacts with an underlying medical condition, please consult with Student Life Disability Services to request reasonable accommodations. You can connect with them at slds@osu.edu; 614-292-3307; or https://slds.osu.edu.

### Religious Accommodations

Ohio State has had a longstanding practice of making reasonable academic accommodations for students' religious beliefs and practices in accordance with applicable law. In 2023, Ohio State updated its practice to align with new state legislation. Under this new provision, students must be in early communication with their instructors regarding any known accommodation requests for religious beliefs and practices, providing notice of specific dates for which they request alternative accommodations within 14 days after the first instructional day of the course. Instructors in turn shall not question the sincerity of a student's religious or spiritual belief system in reviewing such requests and shall keep requests for accommodations confidential.

With sufficient notice, instructors will provide students with reasonable alternative accommodations with regard to examinations and other academic requirements with respect to students' sincerely held religious beliefs and practices by allowing up to three absences each semester for the student to attend or participate in religious activities. Examples of religious accommodations can include, but are not limited to, rescheduling an exam, altering the time of a student's presentation, allowing make-up assignments to substitute for missed class work, or flexibility in due dates or research responsibilities. If concerns arise about a requested accommodation, instructors are to consult their tenure initiating unit head for assistance.

A student's request for time off shall be provided if the student's sincerely held religious belief or practice severely affects the student's ability to take an exam or meet an academic requirement and the student has notified their instructor, in writing during the first 14 days after the course begins, of the date of each absence. Although students are required to provide notice within the first 14 days after a course begins, instructors are strongly encouraged to work with the student to provide a reasonable accommodation if a request is made outside the notice period. A student may not be penalized for an absence approved under this policy.

If students have questions or disputes related to academic accommodations, they should contact their course instructor, and then their department or college office. For questions or to report discrimination or harassment based on religion, individuals should contact equity@osu.edu

#### Additional Policies

 $For more information on university policies, consult \ https://ugeducation.osu.edu/academics/syllabus-policies-statements/standard-syllabus-statements$ 

# Course Syllabus and Schedule

This course consists of 3 parts with 7 chapters.

Part 1: Probability Measures and Integrations

Chapter	Content	References
1 Probability Measures	<ul> <li>σ-algebra</li> <li>Probability measures</li> <li>Random variables</li> </ul>	[4] Ch 1,2,8; [5] Ch 1
2 Integrations and Convergence	<ul><li>Lebesgue integrals</li><li>Convergence of random variables</li><li>Convergence theorems</li></ul>	[4] Ch 9; [5] Ch 1
3 Conditional Expectations	<ul> <li>Information and sub σ-algebra</li> <li>Conditional probabilities</li> <li>Conditional expectations</li> <li>Change of measures</li> </ul>	[4] Ch 3, 10, 23; [5] Ch 4

Part 2: Brownian Motion and Stochastic Calculus

Chapter	Content	References
4 Martingales and Brownian Motions	<ul><li> Markov processes and martingales</li><li> Brownian motions</li></ul>	[1] Ch 3; [3] Ch 2
5 Stochastic Calculus	<ul><li>Constructions of Itô's integrals</li><li>Itô's lemma and applications</li></ul>	[1] Ch 4, [2] Ch 3-4 [3] Ch 3
6 SDEs	<ul><li>General theory of SDEs</li><li>Solving linear SDEs</li><li>Simulations of solutions</li></ul>	[1] Ch 6 [2] Ch 5 [3] Ch 5

Part 3: Black-Scholes Model and Risk-Neutral Pricing

Chapter	Content	References
7 Risk-Neutral	• Risk-neutral measures	[1] Ch 5
Pricing	<ul> <li>Martingale representation theorem</li> <li>Black-Scholes formula</li> <li>Option pricing</li> </ul>	

## Schedule

The following table lists the tentative schedule of the course. Students may expect deviations from the table according to actual teaching progress.

Week	Dates	Topics	HW/Exam
1	8/26 - 8/29	Probability Measure	_
2	9/1 - 9/5	Probability Measure/Integrations	HW1
3	9/8 - $9/12$	Integrations and Convergent Theorems	_
4	9/15 - 9/19	Convergent Theorems/Conditional Expectations	HW2
5	9/22 - $9/26$	Conditional Expectations/Martingales	HW3
6	9/29 - 10/3	Martingales and Brownian Motions	HW4
7	10/6 - 10/10	Itô Integrals	_
8	10/13 - 10/17	${ m Midterm/Autumn~Break}$	Midterm
9	10/20 - 10/24	Itô Integrals and Itô's Lemma	
10	11/3 - $11/7$	Itô's Lemma/SDEs	HW5
11	11/10 - 11/14	SDEs/Risk-Neutral Pricing	_
12	11/17 - 11/21	Risk-Neutral Pricing	HW6
13	11/24 - 11/28	Risk-Neutral pricing/Thanksgiving	_
14	12/1 - $12/5$	Risk-Neutral Pricing	HW7
15	12/8 - $12/12$	Final Review/Exam	Exam

## **Exam Dates**

1. Midterm: TBD (10/8, 10/10, or 10/13)

2. Final Exam: 12/18/2024, Thursday,  $4:00 \mathrm{pm}\text{-}5:45 \mathrm{pm}$  at Enarson Classroom Bldg 245